KARST DE JONG AND THOMAS NOLL

Contiguous Fundamental Bass Progressions

This article documents an intermediate stage of a collaborative work-in-progress, in which theory, musical analysis, and teaching of improvisation are closely intertwined. It is divided into two sections. The first section proposes a new interpretation of fundamental bass progressions, inspired by David Lewin, Ernö Lendvai, Nicolas Meeùs, and others. Here we intend to give an overview and provide some orientational landmarks of theoretical interest. For the fundamental bass progressions, it introduces an arrow notation that highlights the relations *between* fundaments, rather than the fundaments themselves. This arrow notation greatly facilitates practical and pedagogical applications of transformational theory. The second section explores the usage of the arrow notation for what it can add to harmonic analysis and the teaching of improvisation in common practice music, including jazz.

1 Back to the Roots

To a certain degree, roman numerals and functional symbols are useful for the conceptualization of common-practice harmonic tonality. But the lack of a truly working theory – such as physicists have developed in several branches of their field – makes it desirable for musicians, teachers, and theorists to look for alternatives as well. Both systems always force us to label chords relative to a chosen key. With respect to subtle ambiguities in many musical situations, this is often improper. With respect to the constitution of the fundamental bass progression, it constantly masks its essence by an awkward denotation practice.

Our proposal selects several elements from commonly known sources and strands of music-theoretical discourse and recombines them in a convenient way. An arrow notation for the fundamental bass has earlier been proposed by Nicolas Meeùs (2000) along with a graphical notation, called *vector harmonique*. Although we deviate in several respects from Meeùs's design, we were inspired by his analytical attitude to let the fundamental bass speak for itself rather than to load it with the constraints of tonal hierarchies from the outset.

Our arrow-notation for fundamental bass progressions is also inspired by David Lewin's (1987) advocacy for the arrow as an emblem of transformational theory. It thus intends to formalize progressions as musical acts rather than musical objects. It is based on the vertical and horizontal directions of a two-dimensional tone lattice – spanned by perfect fifths and minor thirds. The vertical direction represents *proper progressions* while the horizontal direction represents zero progressions, comparable to *Seitenbewegung* in voice leading or - on a more sophisticated theoretical level - to chromatic alteration. The zero progressions are also called *substitutions*.[1] The following subsection presents the notation system in a straightforward way.

1.1 The Arrows

The two arrows shown in Figure 1 are elementary symbols for simple proper progressions. In our view every single act of a proper progression involves exactly one fifth (up or down) and possibly multiple minor-third substitutions. The diminished fifth is considered as a

1 In a more theoretical introduction with connections to transformational scale theory we investigate a modally refined way of analyzing fundamental progressions where we distinguish between 'fifth down' and 'fourth up'. In this theory the role of the minor third is explained in terms of the group structure of the non-commutative Pythagorean lattice in interaction with pitch height (De Jong and Noll 2007).

double minor third and is therefore notated by double-stroke arrows to the left or right. Figure 2 shows the arrows signifying a fundamental substitution, or *Seitenbewegung*.

Simple dominant progession	Simple subdominant progession
fifth down (or fourth up)	fifth up (or fourth down)

Figure 1
Simple proper progressions.

single minor third down	single minor third up
diminished fifth down	diminished fifth up

Figure 2
Substitutions.

Furthermore, we propose that every other fundamental bass progression be expressed as a compound progression, consisting of the combination of one fifth-progression and one substitution, whereby the fifth-progression may be either followed or preceded by that substitution. This yields 16 possibilities, which are listed in Figure 3. We say that a compound progression is *causal* when the fifth progression precedes the substitution; it is *final* when the progression follows the substitution.

Compound Dominant Progressions		Compound Subdominant Progressions	
Causal order	Final order	Causal order	Final order
M2 up	M2 up	M2 down	M2 down
M3 down	M3 down	M3 up	M3 up
m2 down	m2 down	m2 up	m2 up
A1 down	A1 down	A1 up	A1 up

Figure 3
Compound progressions.

The decision on causal or final order requires additional analytical principles. The situation resembles the orthographical ambiguities in the notation of chromatic steps. Our general preference for the final order parallels the orthographical rule given by Rudolf Louis and Ludwig Thuille (1907, p. 282), which states that chromatic alterations precede diatonic steps.[2] Tonal constraints may of course interfere with this principle and yet suggest causal order.

1.2 Contiguity in the Free P5/m3 Lattice

The basic configuration space for fundamental bass progressions is free and unlimited. We assume that the fundamental bass moves in contiguous pathways. Each single progression goes at most one lattice step down or up. Examples of pathways in this lattice are given in section 2. Figure 4 shows the P5/m3 lattice.

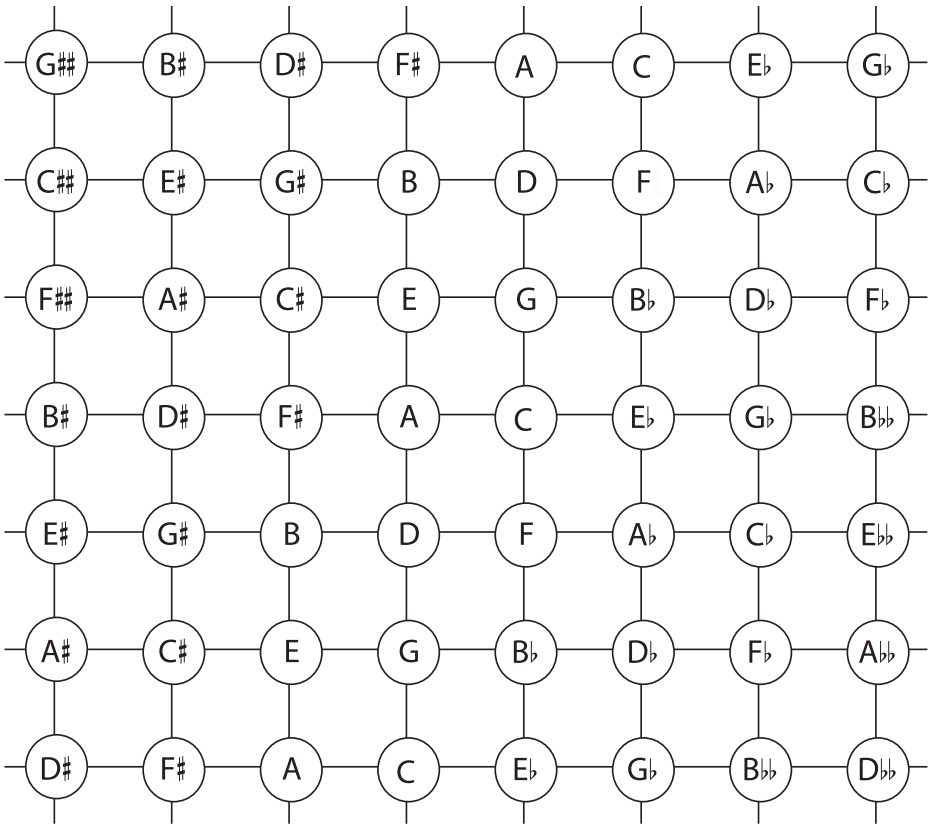


Figure 4
The P5/m3 Lattice.

2 According to Noll, Clampitt, and Dominiguez (2007), the minor third plays an analogous role as a 'generalized comma' in the folding of a four-note scale, like C – F – G – C, as the usual alteration by an augmented prime does in the diatonic C – D – E – F – G – A – B – C.

Formally, the underlying lattice looks the same as Fred Lerdahl's visualization of Jacob Gottfried Weber's space of tonal regions, namely when parallel major/minor regions are completely mixed.[3] However, our interpretation of this lattice differs ontologically from Lerdahl's. For Lerdahl, it represents a collapsed regional space, in which parallel major and minor regions, such as C major and C minor, are fused into single tonal regions. One may interpret the labels at Lerdahl's lattice nodes as chord roots which *pars pro toto* stand for rich musical structures, namely modally mixed regions. In our interpretation, lattice nodes stand for *fundaments* without any further tonal implications.

The space spanned by fifth and minor third looks similar to the space spanned by fifths and major thirds studied by Leonard Euler, Arthur von Oettingen and – last but not least - Hugo Riemann. In particular, transformational theory (see subsection 1.3) developed alongside the investigation of the Euler/Riemann *Tonnetz*.

Remark: While acknowledging the structural coincidence of our basic configuration space for fundamental bass progressions with Lerdahl's collapsed regional space, we do not intend to conflate the meanings of fundament and collapsed tonal region. In his critique of Hugo Riemann's ontologically different interpretations of the *Tonnetz* (intervals, chord relations, regional relations), Lerdahl argues that theorists should not expect a single model to work simultaneously at all levels.[4] The elementary status of the minor third in Lerdahl's reading is motivated by the regional neighbourhood of relative major and minor regions. The elementary status of the minor third in our approach, however, is motivated empirically by our fundamental bass analyses. It is furthermore motivated by an observation in *transformational mode theory* (see footnotes 1 and 2). For the moment it remains an open question whether there is a systematic connection between the different roles of the minor third in harmonic tonality.

1.3 Fundamental Bass Progressions and Tonality

The assumption of a free unlimited space for the progression of the fundamental bass is not realistic for an advanced theory of harmonic tonality. We know that tonal harmony constrains the routes taken by the fundament. But little is known about the principles that govern these constraints. On the one hand, in scale degree theory the scale steps come 'out of nowhere' and the triads and seventh chords are constructed as third-chains (with thirds as double-steps). On the other hand, in functional harmony the primary triads come out of nowhere (at least the minor ones), and the diatonic scale is a motley collection of triad tones. With such a poor understanding of tonality it is not possible to understand the mechanisms of tonal deflections and modulations. Although it is desirable to make tonally sensitive fundamental-bass analyses, it is a disembarrassment for the analyst to have a tonally free neutral level of description, where the path of fundaments may be traced without the constraints of bulky theories of harmonic tonality.

The concept of fundamental bass progression is first of all embedded in Jean-Phillippe Rameau's comprehensive and much-disputed theoretical work. Our deliberate decision to start with an isolated investigation of the fundamental bass progression, and not to consider counterpoint, chord structure, diatonicity, etc., implies the necessity to make a detour from Rameau's original ambitious programme. Especially, the exciting connection between the fundament progression and the chain-linking of dissonances is a subject we have to leave for a future investigation. Nicolas Meeùs (2000) convincingly argues in

3 Lerdahl 2001, p. 112, Figure 3.23 b. Strictly speaking, Lerdahl's visual representation lacks edges between the nodes. This is in accordance with the fact that he conceives of the Weber space as a metric space rather than a graph. However, as an auxiliary construction for the definition of this metric, Lerdahl does in fact use a graph that involves more edges.

4 Lerdahl 2001, p. 45.

favour of the fundamental bass as a level of description in its own right. His proposed graphical representation *vector harmonique* combines the notation of fundament positions and fundament progressions. This visually exposes the progressions as such, and thereby highlights aspects of harmony that are submerged in the traditions of scale degree theory and functional harmony. Both traditions pay much attention to the chords or functions in the sense of positions within systems of harmonic signifiers or signifieds, and suffer from a lack of attention for the harmonic *process*. As is generally known, both traditions spring from different aspects of Rameau's network of ideas. A deliberate concentration on the fundamental bass may help integrate current music-theoretical knowledge into a programme that recaptures the spirit of Rameau.

1.4 Transformational Aspects

One misconceives the nature of progressions if one understands them merely as successions of harmonic positions (e.g. scale degrees or tonal functions). In addition to the sequential structure it is worth paying attention to the concatenation of progressions. David Lewin and other authors in the recent Neo-Riemannian tradition emphasized that mathematical group theory offers suitable means to grasp the underlying music-theoretical facts. Hugo Riemann himself adapted Euler's and Oettingen's decomposition of musical intervals into octaves, fifths, and major thirds to a notation technique for *Tonverwandtschaften*. Edward Gollin (2000), who carefully traces the history of ideas of transformational theory, acknowledges the implicit transformational content in Riemann's notation of *Tonverwandtschaft*.[5]

Our fundamental bass notation follows pretty much the technique of Neo-Riemannian analyses, where triadic chord progressions are analysed in terms of *Schritt/Wechsel* transformations (expressed with letters such as R, L, P, or sometimes D). The commonality is at least twofold: (a) the analytical notation is relative, i.e. it refers to progressions rather than chords or harmonic positions, and (b) there is a debate about basic and compound transformations, relative to a chosen basis. Aside from these formal commonalities there are concrete connections with the music-theoretical content. As long as triads are concerned, our approach relates to a generation of the *Schritt/Wechsel-Gruppe* by the *Quintschritt* D and the *Terzwechsel* R. The deliberate interest of Neo-Riemannian investigations in the *Leittonwechsel* as a potentially elementary transformation in certain romantic compositions can be envisaged as a challenging point of theoretical tension with respect to our proposed approach. The parsimonious leading-tone exchange of the Riemann roots (e.g. C moving to B if we have a C-Major Triad C+ followed by an E-Minor triad E-) corresponds to a major third progression in the fundamental bass (i.e. from C to E in the considered case). With our arrows we would decompose this progression into a fifth up and a minor third down or vice versa, as shown in Figure 5.

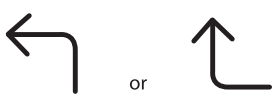


Figure 5
Progression of a major third up.

This theoretical tension parallels to a certain extent the tension between Ernö Lendvai's original and exclusive design of his axial system for harmonic functions,[6] and Richard Cohn's complementary formulation in his analysis of the first movement of Schubert's B-flat Major

5 See for further reading also the introductory article by Henry Klumpenhouwer (2000) in this Journal.

6 Lendvai (1995, 35) asserts that the 'model 1:3' (his term for the hexatonic system) neutralizes the sense of tonality, while the models 1:2 and 1:5 have a distinct tonal character.

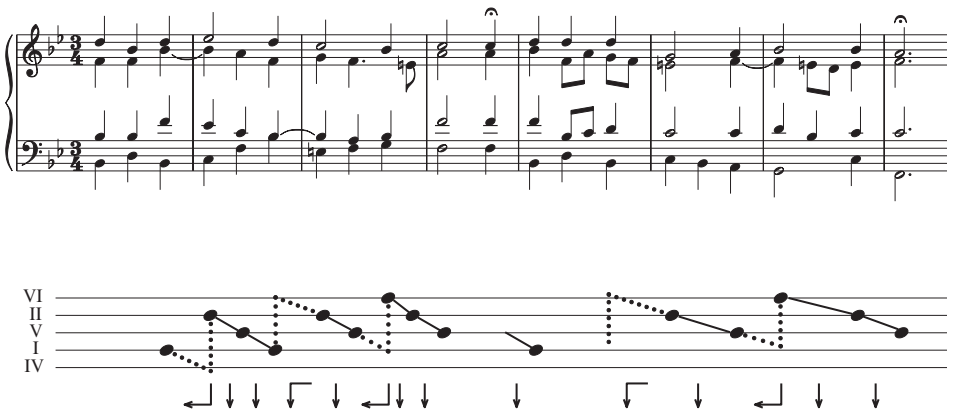
Sonata, which features major thirds as functional substitutes. Our approach is more in line with Lendvai's. However, we do not consider harmonic functions, and our concept of substitution is not burdened with the disputed concept of functional equivalence.

1.5 On Substitutions

In the present paper we introduce the concept of substitution axiomatically: perfect fifths are classified as proper fundament progressions, while minor thirds are considered as substitutions of the fundament. In addition to the Riemannian *Terzwechsel* and Lendvai's radical design of the axial system within a twelve-tone chromatic there is also a non-functional concept of substitution in Rameau: the double employment. Rameau postulates that the fundament has to progress in consonant intervals: fifth and thirds and their inverses. There are several exceptions that seem to violate his postulate: I → II, IV → V, and V → VI. The double employment is a contrapuntal argument that explains a substitution in the fundament in terms of a substitution of the dissonant interval (the dissonant added sixth is substituted by a seventh). It works fine for the first two cases and not so good if the V → VI is a deceptive cadence. Yet, in our simplified model, we assume that in such cases the fundamental bass is substituted by a bass a minor third below. The deceptive cadence in minor is a different case in our approach, and is discussed in De Jong and Noll 2007.

Our usage of the concept of substitution is closely related but – strictly speaking – not identical to Meeùs's approach. It differs in the following detail. When we speak of substitution, the substituted entity is a fundament. Meeùs applies the concept of substitution also to a higher level of description. In his view, third- or second-progressions are substitutions for fifth-progressions. The substituted entity is then a progression. Both readings become related as soon as the substitution component in a compound progression is neglected in favour of its progression component. Aside from these formal differences, there are differences in the classification of major thirds and minor seconds, which require a ramified discussion.[7]

Meeùs does not explicitly mention the problem of order in compound progressions (such as the double employment). But in his analyses he is confronted with the need for a decision in each case. Example 1 translates Meeùs's analysis of the first two lines of the Bach Chorale, BWV 321 into our arrow language. There are three I-II successions and two V-VI successions, which are analysed as compound progressions.



Example 1
J.S. Bach Chorale BWV 321.

7 Meeùs would decompose upward minor seconds in analogy to upwards major seconds, i.e. as a major third down followed by a fourth up (or vice versa). As we allow only minor third substitutions, we would instead decompose the upward minor second into a fourth down followed by a diminished fifth up (or vice versa).

As we see in his analysis, Meeùs chooses three times the *causal order* where the progression comes first and is followed by a substitution; and he uses twice the *final* order, where the substitution occurs first and is followed by a progression. We cannot be sure that he reflected upon these decisions.

2 Analysis and Improvisation

To understand what the fifth progressions, minor third substitutions and their compound progressions mean in tonal music, our initial approach was to identify and describe them. In order to do so, we analysed pieces from the literature in the widest possible range, from Sweelinck to Coltrane. While doing that, we noticed large differences in the frequency with which various progressions and combinations of progressions occur, depending on style. In music of the common-practice era for example, *down* and *left* progressions are far more common than *up* and *right* progressions. When looking at series of progressions (arrow patterns), it also became apparent that in certain style periods certain arrow patterns occur very often, while others are rare.

As already remarked in section 1.2, we initially consider the fundaments to exist on their own, without any tonal obligations, constraints, or further implied meaning. For the moment, we do not define any rules for the quality of upper structures, and leave this to future investigation. However, we realize that in tonal music upper structures are necessary to create a convincing sense of tonality. How independent can fundament progressions be? And if they are not completely independent, what is their relation to these upper structures? We think that improvisation can help us understand this relation, as it is an outstanding tool for exploration. In the following paragraphs we show and discuss some of our analytical findings, and suggest a number of improvisation exercises.

2.1 Circles of fifths (down and double-left only)

The first pattern to take a closer look at is the diatonic circle of fifths. Being omnipresent in the musical repertoire, it offers a good starting point for exploration by improvisation. Figure 6 shows the typical patterns for major and minor keys:

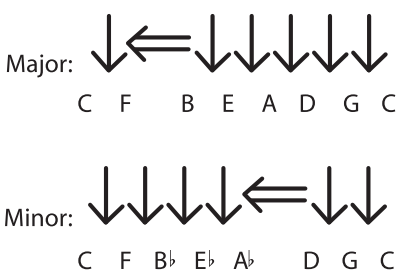


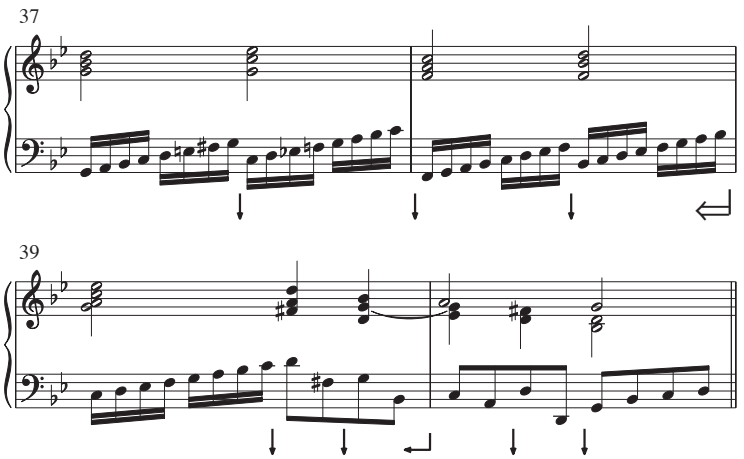
Figure 6
Diatonic circle of fifths.

Each pattern consists of six *downs* and a *double-left*, the latter being necessary to close the diatonic circle.

Improvisation exercise 1: Play a free improvisation on the fundamental bass patterns of the diatonic circle. Use as much variation as possible in the choice of chords, texture, metre and metrical placement. Experiment with the place of the *double-left* in the circle. Expand the exercise by using *downs* and *double-lefts* freely. The exercise can be expanded in many ways, such as starting on different places in the pattern, experimenting with metre, etc.

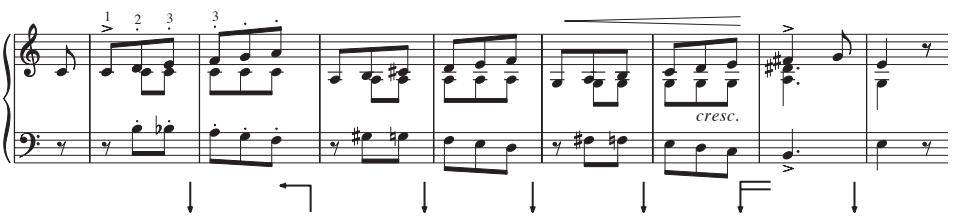
The suggested improvisation exercises are meant to be played in a tonal idiom. Since we say nothing in our theory about the connection of the fundamental bass and the upper structures (or chords), all responsibility of chord choice lies with the improviser, who is thus forced to develop an ability to play logical upper structures on the given fundamental bass patterns. The freedom in choice of upper structures should stimulate an intuitive and critical approach. It will soon become clear that alternative non-diatonic chords work well on most fundaments. Elements of voice leading can so be addressed without an all-too dogmatic approach of which chord to play on which bass note.

As we noticed in some examples from the literature, the *double-left* and *down* sometimes merge into a compound progression. Example 2 shows an excerpt from Handel's *Passacaille* (Keyboard Suite, No. 7) where this happens at the end of the second measure. Effectively, this results in a 'skip' of the tritone in the pattern. This is consistent with our assumption that one of the two fundaments involved in a diminished fifth can be left out. In other words, the diminished fifth behaves as a substitutional progression.



Example 2
Händel, Passacaille from the Keyboard Suite, No.7 Skipping the tritone in the circle.

A similar skip occurs in Schumann's *Papillons* (Example 3). The half step down between mm. 6 and 7 can be analysed as a *double-left-down*. Seen from the goal of E minor, the fundament C is followed directly by the dominant B, without the intervention of F♯.



Example 3
Schumann, *Papillons*, No. 10.

2.2 The footprint pattern

When we consider the full chromatic circle, the diatonic circle can be regarded as a shortened version of it, whereby the *double-left* substitution functions as a shortcut. The restriction to seven fundaments creates a 'focus', which is related to the traditional concept of tonality. When we focus even more, creating a circle with just four fundaments, we find a pattern, which from now on we will call *footprint.* In this footprint pattern, a *left* substitution is compensated by three *downs* (Figure 7).

Figure 7

We find numerous examples of this pattern in all musical styles. To name a few: The basic jazz-turnaround, Beethoven's 'Spring' Sonata (first movement, mm.1-9), Schumann's *Papillons*, No.4 (mm.1-9), Monty Python's *Always Look at the Bright Side of Life* (chorus), and Monk's *Round Midnight* (a good example of this pattern in a minor tonality!).

Sometimes the first *left* and *down* are merged into a compound *left-down* progression as in Figure 8. Now the circle is even further shortened, including just three fundaments. This pattern often occurs at the beginning of a piece. The tonic is followed by a second degree, either in root position or as an inverted seventh chord, and the dominant. Some examples are: J.S. Bach, the C-major prelude from *The Well-Tempered Clavier*, Book I (beginning), Schumann's *Kinderszenen*, No. 1 (beginning), Handel's aria *Lascie ch'io piange* (theme).

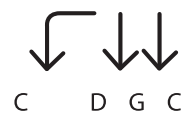


Figure 8

Improvisation Exercise 2: Explore the footprint pattern in different tonalities, both major and minor. Try to use as many chord-forms on it as possible. Experiment also with different permutations of the arrows, like the one in Figure 9. In a later stage, replace simple progressions by compound ones, where possible.

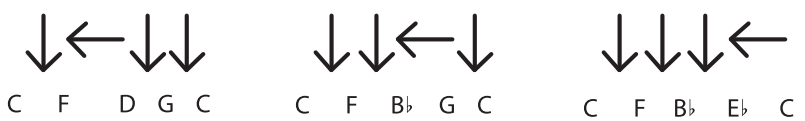


Figure 9

The permutation of a series of arrows is in itself a very interesting phenomenon. On the one hand, the goal remains the same, but the road leading there is different. Some permutations lead to mere 'changes' in the harmonization, others may affect the tonality and even the tonal meaning of the final goal. Experimenting with these permutations in improvisation can be a powerful way to explore tonality.

2.3 Rhythm Changes

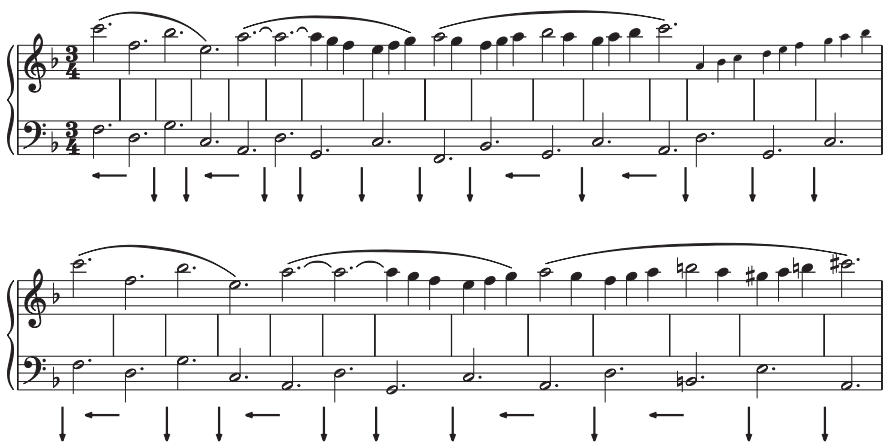
The footprint pattern can be extended by embedding other footprint patterns, thus prolonging one of the involved fundaments. A common example is one where the dominant is extended by replacing the expected *down* progression to the tonic with a *left* substitution to the third. The resulting pattern is used for instance in the so called 'rhythm changes' (Figure 10).

Figure 10

Improvisation exercise 3: Improvise with free combinations of *left* and *down* progressions. Flow freely through different tonalities. Experiment with the prolongational possibilities of the footprint pattern.

2.4 Waltz For Debby

The composition *Waltz for Debby* by Bill Evans suggests further ways to extend musical structures with the footprint pattern. Evans prolongs the tonal tension by using only *left* and *down* connections. Example 4 shows a reduction of *Waltz for Debby*. In the second staff, instead of the sounding bass the fundamental bass is written with arrows underneath.



Example 4
Bill Evans, *Waltz for Debby*, mm. 1-30.

Toward the end of each phrase we see a quicker alternation of *left* and *down* progressions, creating a sense of immediacy and raising tension.

2.5 Up and Down: neighbour progressions

Figure 11

The *up* (or subdominant) progression is often immediately followed by a *down*, resulting in a neighbour progression (Figure 11). In many beginnings of classical compositions it serves to prolong the tonic. Examples of this can be found in the 'Easy' Sonata (second movement, mm.1-8) and Sonata 'Pathetique' (second movement, mm.1-4) by Beethoven, and in the Sonata in C minor, K. 457 by Mozart (first movement, mm.1-8). The subdominant progression may also serve as a prolongation of any other given fundament than the tonic. Some pieces start with a few subdominant progressions in a row, building up tension in an alternative way. Examples of this can be found in Schubert's String Quintet (second movement, mm.1-6) and Bach's Prelude in B minor from *The Well-Tempered Clavier*, Book I (mm.1-2).

Improvisation exercise 4: Improvise a theme that consists only of alternating up and down progressions. As a next step, use the *up-down* pattern to make prolongations of other chords within a given harmonic structure.

2.6 Right progressions

The *right* progression seems to be the least-used progression in the common-practice styles. The opposite of the left progression, it functions as a shortcut to the fundament that lies three fifths down. Consequently, it can be compensated by three up progressions (Figure 12, left). We find examples in early music as well as in romantic, impressionistic and jazz/pop styles. A few examples that employ the *up-right* pattern (Figure 12, right): Sweelinck, *Chromatic Fantasia*, mm.6-9, Beethoven, 'Waldstein' Sonata, mm.1-8, and Chopin, *Polonaise-Fantasie* (theme). By harmonizing this sequence with major chords a fascinating descending chromatic line emerges in the upper voices.

Figure 12

In all examples mentioned above, the *right* is partly compensated by one or more *up* progressions. When a sequence is made of *right-down* progressions, however, the result sounds much more 'modern'. Examples of this can be found as early as in Schubert, but a particularly interesting one is found at the climax of Ravel's *Ondine*, from the cycle *Gaspard de la Nuit*, in mm.67-68 (Figure 13). The full chromatic circle is completed in six progressions. John Coltrane appears to have been deeply inspired by this piece of Ravel: in *Giant Steps* he not only used a similar pattern (also starting on B!), but also borrowed melodic material and the phrasing from Ravel. Example 5 shows the similarities.

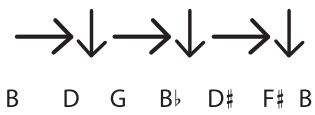
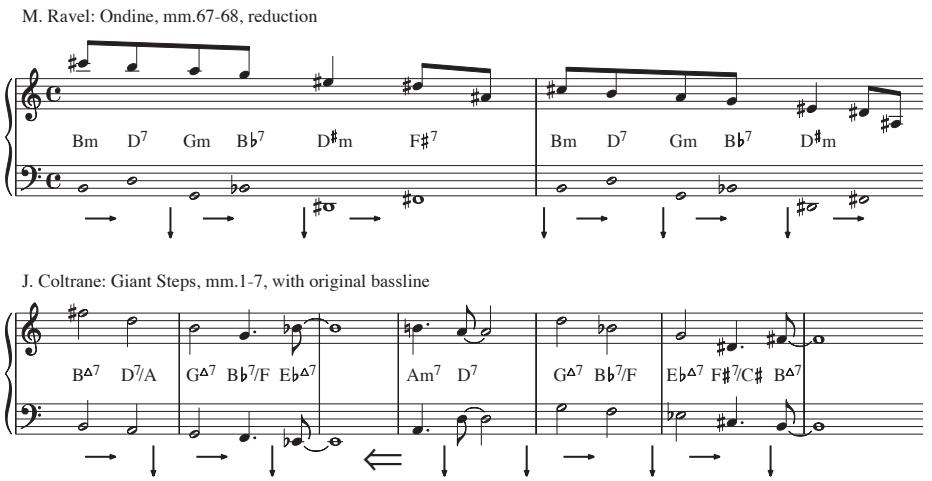


Figure 13



Example 5
M. Ravel, *Ondine* and J. Coltrane, *Giant Steps* compared.

2.7 Abstract Patterns

When viewed in the free P5/m3 lattice, the Ravel/Coltrane example looks like a pattern going down the 'stairs' from top-left to bottom-right. It is great fun to improvise along this and other abstract patterns within the lattice. Figures 14, 15, and 16 show some other possibilities: 'The Square', 'The Castle Wall' and 'The "Monte" Sequence'. Using a bit of imagination, many more can be thought of and experimented with.

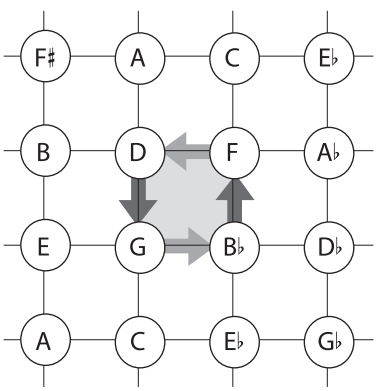


Figure 14
"The Square": this pattern occurs for example in the coda of the Beatles' *Yesterday*.

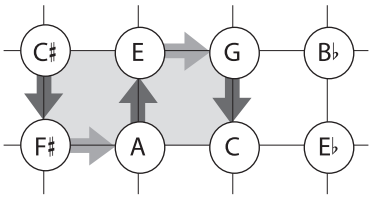


Figure 15
"The Castle Wall": the fundaments form an octotonic set.

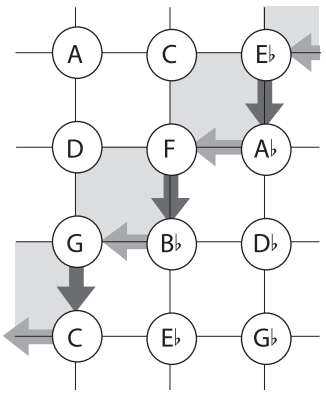


Figure 16
"'The "Monte" sequence'": dominant direction.

Conclusions

So far, our assumption that a contiguous motion through the P5/m3 lattice underlies all possible fundament progressions seems to be at least promising. The arrow notation provides a simple and convenient way of labelling and communicating these progressions, effectively showing the degree of complexity of harmonic relations and relational patterns. Consequently, we believe that the arrow symbols have a great potential for practical and didactical applications.

The interpretation of fundament progressions in the P5/m3 lattice provides an alternative approach to the study of tonality, possibly revealing a 'reptile brain' of tonality: progressions following a path of fundaments in a world governed by footprint patterns, where fourths, fifths, minor thirds, and major seconds rule, and where minor seconds and major thirds are indirectly reached. A refined theoretical approach should explain the particular role of the minor third as a substitutive interval.

For the near future, we hope that the arrow notation and the general chain of thoughts in this article will be taken up by colleague theorists and musicians, and stimulate further thinking in diverse areas of music theory and music teaching. Particularly, it will be challenging to conduct integrated empirical investigations into the interaction of fundamental bass progressions, voice-leading, and musical form, through analysing entire musical corpora.

Bibliography

Cohn, Richard (1999) 'As Wonderful as Star Clusters: Instruments for Gazing at Tonality in Schubert', in: *Nineteenth-Century Music* 22/3, 213-232.

De Jong, Karst, and Thomas Noll (2007), 'The Fundamental Path: Towards an Integration of Theory, Analysis, and Teaching of Fundament Progressions' (lecture, Koninklijk Conservatorium, Den Haag, June 26, 2007).

Gollin, Edward (2000) *Representations of Space and Conceptions of Distance in Transformational Music Theories*, PhD Dissertation, Harvard University.

Klumpenhouwer (2000), 'Remarks on American Neo-Riemannian Theory', in: *Dutch Journal of Music Theory* 5/3, 155-169.

Lendvai, Ernö (1995) *Symmetrien in der Musik: Einführung in die musikalische Semantik,* Kecskemét: Kodály Institut, Vienna: Universal Edition.

Lerdahl, Fred (2001) *Tonal Pitch Space*, New York etc.: Oxford University Press.

Lewin, David (1987) *Generalized Musical Intervals and Transformations*, New Haven: Yale University Press.

Louis, Rudolf, and Ludwig Thuille (1907) *Harmonielehre.* Stuttgart: Ernst Klett.

Meeùs, Nicolas (2000) 'Toward a Post-Schoenbergian Grammar of Tonal and Pre-tonal Harmonic Progressions', in: *Music Theory Online* 6/1.

Noll, Thomas, David Clampitt and Manuel Dominguez (2007) 'What Sturmian Morphisms Reveal about Musical Scales and Tonality'. Paper presented at WORDS 2007: Sixth International Conference on Words, Marseille, September 17-21, 2007.

Rameau, Jean Phillipe (1722) *Treatise on Harmony*, English Translation by Philip Gossett. New York: Dover Publications, 1971.