Oplossingen Hoofdstuk 6

Hieronder staan oplossingen van enkele vragen uit Hoofdstuk 6.

1.

- Lengte van de vectoren:

$$\begin{aligned} \|A_1\| &= 1 \\ \|A_2\| &= \sqrt{5} \\ \|A_3\| &= \sqrt{14} \\ \|A_4\| &= 2 \end{aligned}$$

- Inproduct tussen elk tweetal vectoren:

$$\begin{aligned} \langle A_1, A_2 \rangle &= 2 \\ \langle A_1, A_3 \rangle &= 3 \\ \langle A_1, A_4 \rangle &= -1 \\ \langle A_2, A_3 \rangle &= 8 \\ \langle A_2, A_4 \rangle &= -1 \\ \langle A_3, A_4 \rangle &= 0 \end{aligned}$$

- Hoeken tussen elk tweetal vectoren:

$$\begin{aligned} \theta(A_1, A_2) &= \arccos(2/\sqrt{5}) \\ \theta(A_1, A_3) &= \arccos(3/\sqrt{14}) \\ \theta(A_1, A_4) &= 2\pi/3 \\ \theta(A_2, A_3) &= \arccos(8/\sqrt{70}) \\ \theta(A_2, A_4) &= \arccos(-1/(2\sqrt{5})) \\ \theta(A_3, A_4) &= \pi/2 \end{aligned}$$

- Afstanden tussen elk tweetal vectoren:

$$\begin{aligned} d(A_1, A_2) &= \sqrt{2} \\ d(A_1, A_3) &= 3 \\ d(A_1, A_4) &= \sqrt{7} \\ d(A_2, A_3) &= \sqrt{3} \\ d(A_2, A_4) &= \sqrt{11} \\ d(A_3, A_4) &= 3\sqrt{2} \end{aligned}$$

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2.

- Lengte van de vectoren:

$$\begin{aligned}\|p_1(X)\| &= \sqrt{8}/\sqrt{3}\\ \|p_2(X)\| &= 2\sqrt{14}/\sqrt{15}\\ \|p_3(X)\| &= 4/\sqrt{15}\end{aligned}$$

- Inproduct tussen elk tweetal vectoren:

$$\begin{aligned}\langle p_1(X), p_2(X)\rangle &= 8/3\\ \langle p_1(X), p_3(X)\rangle &= 4/3\\ \langle p_2(X), p_3(X)\rangle &= 16/15\end{aligned}$$

- Hoeken tussen elk tweetal vectoren:

$$\begin{aligned}\theta(p_1(X), p_2(X)) &= \arccos(\sqrt{5}/\sqrt{7})\\ \theta(p_1(X), p_3(X)) &= \arccos(\sqrt{5}/\sqrt{8})\\ \theta(p_2(X), p_3(X)) &= \arccos(\sqrt{2}/\sqrt{7})\end{aligned}$$

- Afstanden tussen elk tweetal vectoren:

$$\begin{aligned}d(p_1(X), p_2(X)) &= 4/\sqrt{15}\\ d(p_1(X), p_3(X)) &= 4/\sqrt{15}\\ d(p_2(X), p_3(X)) &= \sqrt{8}/\sqrt{3}\end{aligned}$$

4. $\{(-2\lambda, \lambda, 2\lambda) \mid \lambda \in \mathbb{R}\}$

5. Een mogelijke oplossing is $\left\{\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right), \left(\frac{4}{3\sqrt{5}}, \frac{-2}{3\sqrt{5}}, \frac{5}{3\sqrt{5}}\right), (-\frac{2}{3}, \frac{1}{3}, \frac{2}{3})\right\}$.

6. Een mogelijke oplossing is $\left\{1, \sqrt{3}(2x-1), 6\sqrt{5}\left(x^2 - x + \frac{1}{6}\right)\right\}$.

7. Een mogelijke oplossing is $\left\{\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right\}$.

8.

(a) Een mogelijke schrijfwijze is $U_1^\perp = \mathrm{vct}\{(1, -1, 0)\}$.

(b) Een mogelijke schrijfwijze is $U_2^\perp = \mathrm{vct}\{(1, 0, -1), (0, 1, -1)\}$.

(c) Een mogelijke schrijfwijze is $U_3^\perp = \mathrm{vct}\left\{\begin{pmatrix}-3 & 2\\ -1 & 2\end{pmatrix}\right\}$.

11. Een mogelijke oplossing is $\alpha = \left\{(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}), (-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})\right\}$.

Dan is $T_\alpha^\alpha = \begin{pmatrix}1 & 0\\ 0 & 6\end{pmatrix}$ en $P = \begin{pmatrix}\frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}}\\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}}\end{pmatrix}$.

12. $a \in \{-1, 1\}$

14. Mogelijke oplossingen zijn

$$P_{A_1} = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix} \quad P_{A_2} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \quad P_{A_3} = \begin{pmatrix} \frac{2}{3} & \frac{1}{\sqrt{5}} & \frac{-4}{3\sqrt{5}} \\ \frac{1}{3} & \frac{-2}{\sqrt{5}} & \frac{-2}{3\sqrt{5}} \\ \frac{2}{3} & 0 & \frac{5}{3\sqrt{5}} \end{pmatrix}.$$

15. Neem bijvoorbeeld $\alpha = \left\{ \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, 0 \right), \left(0, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right), \left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) \right\}$.

16.

- $a = 1$: $\dim_{\mathbb{R}}(U^{\perp} + W) = 2$.
- $a = -1$: $\dim_{\mathbb{R}}(U^{\perp} + W) = 2$.
- $a \notin \{-1, 1\}$: $\dim_{\mathbb{R}}(U^{\perp} + W) = 3$.

17.

(a) Nee.

(b) Een mogelijke basis is $\left\{ \left(\frac{1}{2}, 0, \frac{1}{2}, 0 \right), \left(\frac{3}{2\sqrt{19}}, 0, -\frac{1}{2\sqrt{19}}, \frac{2}{\sqrt{19}} \right) \right\}$.

19. Mogelijke oplossingen zijn:

- $$B_1 = U\Sigma V^T = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{6} & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}^T.$$
- $$B_2 = U\Sigma V^T = (-1) \begin{pmatrix} 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} -\frac{2}{3} & \frac{1}{\sqrt{2}} & -\frac{1}{3\sqrt{2}} \\ -\frac{1}{3} & 0 & \frac{4}{3\sqrt{2}} \\ \frac{2}{3} & \frac{1}{\sqrt{2}} & \frac{1}{3\sqrt{2}} \end{pmatrix}^T.$$
- $$B_3 = U\Sigma V^T = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{5}} & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{2}} & 0 & -\frac{2}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{2}} & 0 & -\frac{2}{\sqrt{10}} \\ \frac{\sqrt{2}}{\sqrt{5}} & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}^T.$$

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